Lsn 9-2: Exponential Decay

OBJECTIVES

- recognize properties of exponential functions
- apply exponential decay models
- graph exponential functions

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* in population growth problems, $b > 1$

→ sometimes a growth factor is between 0 & 1, when this happens the value of the function decreases - EXPONENTIAL DECAY

DEPRECIATION AS AN EXAMPLE OF EXPONENTIAL DECAY

* cars & other goods often decrease in value over time - this decrease is called

DEPRECIATION

- if the decrease is r% annually, then each year the item is worth $(100-r)\%$ of its previous value $\quad b = 1 - r$

EX → In 1988, a new Firebird Trans Am cost $15,798, suppose the car depreciates 13% each year, predict the car's value in 2013.

$$y = ab^x$$

$$y = 15{,}798 \cdot (.87)^{25}$$

$$= \$485.94$$

↳ is this accurate?

RADIOACTIVE DECAY AND HALF-LIFE

* amount of radioactive substance decreases over time

→ half-life of a substance is the amount of time it takes for half of the material to decay

EX → Sr-90 has a half-life of 29 years. If you start with 1000g, how much will remain after 290 years?

$$y = ab^x$$

- a: original amount
- b: .5 always for half-life
- x: # of half-lives

$y = 1000(.5)^{10}$ — 290 yrs

$y = .9766\,g$

29 yrs per h.l.
= 10 half-lives

EX → C-14 has a half-life of 5730 yrs, if an artifact has only 20% of the original amount, about how old is it?

$y = ab^x$

$20 = 100(.5)^x$ — # of 5730 yr periods

20 ↓ % 100 ↓ %

graph & find intersection

$x = 2.32$ half-life periods

2.32×5730

about 13,293.6 yrs old

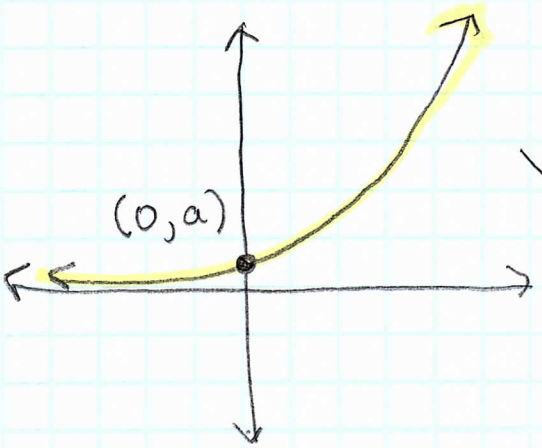
GROWTH VS. DECAY

Exponential Growth Model

If a positive quantity a grows by a factor b ($b > 0, b \neq 1$) in each unit period, then after a period of length x, there will be ab^x of the quantity.

exponential growth

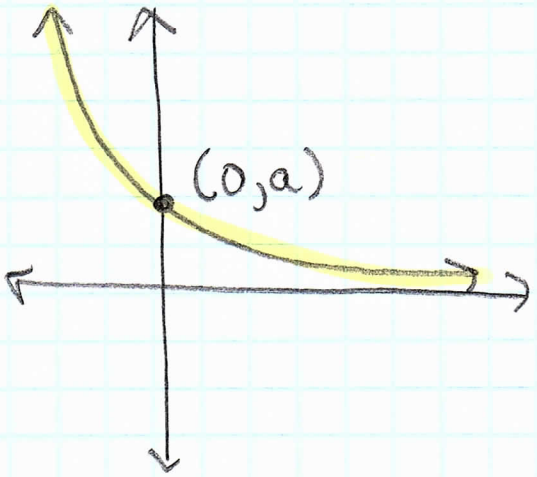
$b > 1$



$y = ab^x$

exponential decay

$0 < b < 1$



* Exponential functions have the following properties:

1) domain – $\mathbb{R}$
2) range – positive $\mathbb{R}$
3) y-int of each graph is a
4) graphs never cross x-axis
5) graphs get closer & closer to x-axis, the asymptote
6) when $b > 0$, function increases
   when $0 < b < 1$, function decreases

EX → Prove $f(x) = 3^{-x}$ is an exponential decay ftn.

$3^{-x} = (3^{-1})^x = \left(\frac{1}{3}\right)^x$

↳ between 0 & 1, decay ✓